

DEPARTMENT OF MATHEMATICS
COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Professor Wazir Hasan Abdi Memorial Lecture - 2019
(Twentieth in the series)

by

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On

20/09/2019

Schedule : *Talk 1 : 2.15 pm – 3.15 p.m., Talk 2 : 3.30 pm – 4.30 p.m.*

Topic: *Entrywise functions preserving positivity: from metric geometry, to covariance estimation, to a new graph invariant*

Talk 1: *Schoenberg: from metric geometry to matrix positivity*

We present a historical account of some work of Schoenberg in metric geometry: from his metric space embeddings into Euclidean space and into spheres (Ann. of Math. 1935), to his characterization of positive definite functions on spheres (Duke Math. J. 1942). It turns out these results can be viewed alternately in terms of matrix positivity: from appearances of (conditionally) positive matrices in analysis, to the classification of entrywise positivity preservers in all dimensions.

Talk 2: *The critical exponent of a graph*

Given a graph G , let P_G denote the cone of positive semidefinite (psd) matrices, with non-negative entries, and zeros according to G . Which powers preserve psd-ness when applied entrywise to all matrices in P_G ?

We present modern motivations to study this question, arising out of high-dimensional covariance estimation -- these are in parallel to the classical origins of positivity preservers described in the preceding lecture. This is followed by the first result for the above question: by FitzGerald and Horn (J. Math. Anal. Appl. 1977), for G a complete graph. We then discuss recent joint work with D. Guillot and B. Rajaratnam (JCT-A 2016), which relates powers preserving positivity to the geometry of the graph G , and leads us to propose a novel graph invariant: the "critical exponent" of G . We compute this critical exponent for all chordal graphs, cycles, and bipartite graphs, and end with some open questions.

Venue: Auditorium, Department of Mathematics, CUSAT

Head of the Department