Functional analysis, Operator algebras, and Noncommutative geometry

Dr. Tathagata Banerjee research focuses on the interface of functional analysis, operator algebras, and noncommutative geometry, where we explore the mathematical foundations of quantum theory and generalized geometrical structures. Our work utilizes advanced techniques from operator theory to study problems arising in quantum information science and the geometry of noncommutative spaces.

One significant aspect of my research involves analyzing the C*-**extreme points of positive operator-valued measures and unital completely positive maps**, which are central to understanding quantum measurements and quantum channels. In this direction we study the C*-convexity structure of positive-operator valued measures as an extension of the C*-convexity structure on the space of all unital completely positive maps from C(X) to bounded operators on a Hilbert space. We show that the C*-extreme points in the normalized case are only the projection-valued spectral measures. Using the correspondence between regular positive operator-valued measures and unital completely positive maps, we also classify the C*-extreme points of unital completely positive maps on commutative C*-algebras with countable spectrum and taking values in the algebra of bounded operators on a Hilbert space. The significance of this work is that we do it even for infinite-dimensional separable Hilbert spaces. We also prove density theorems in line with the famous Krein-Milman theorem in the above-mentioned settings. In another direction of my research we developed a certain **coarse geometric study for noncommutative spaces**, extending classical notions of coarse geometry to spaces where traditional point-based intuition no longer applies, such as in the mathematical models of quantum spaces. Noncommutative spaces are spaces that cannot be described by usual points and distances but are defined in terms of a noncommutative C*-algebra. We develop coarse geometry for noncommutative spaces by defining what it means for them to look similar at large scales in terms of unitizations of non-unital C*-algebras. Using this framework, we studied examples like the Moyal plane, a well-known noncommutative space, and showed that it is equivalent, in a coarse geometric sense, to the ordinary plane.These contributions help in understanding the algebraic and geometric properties of noncommutative and quantum spaces, with broader implications for both theoretical physics and quantum information theory.